Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2J: Further Mechanics 1 and Further Mechanics 2

Sample Assessment Material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference **8FM0/2J**

You must have: Mathematical Formulae and Statistical Tables, calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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SECTION A

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 1 continued

(Total for Question 1 is 10 marks)

2. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of $25\,\text{m}\,\text{s}^{-1}$. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

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Question 2 continued

(Total for Question 2 is 6 marks)

3. [*In this question use* $g = 10\,\text{m}\,\text{s}^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,\text{m}\,\text{s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\text{m}\,\text{s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\text{m}\,\text{s}^{-1}$.

(5)

Question 3 continued

(Total for Question 3 is 8 marks)

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 16 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

5. A particle P moves on the x-axis. At time t seconds the velocity of P is $v\,\text{m}\,\text{s}^{-1}$ in the direction of x increasing, where

$$v = (t - 2)(3t - 10), \qquad t \geqslant 0$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Find the total distance travelled by P in the first 2 seconds of its motion.

(3)

(c) Show that P never returns to O, explaining your reasoning.

(3)

Question 5 continued

(Total for Question 5 is 8 marks)

6. A light inextensible string has length $7a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 5a$. A particle of mass m is attached to a point P on the string where $AP = 4a$. The particle moves in a horizontal circle with constant angular speed ω, with both AP and BP taut.

(a) Show that

(i) the tension in AP is $\frac{4m}{25}(9a\omega^2 + 5g)$

(ii) the tension in BP is $\frac{3m}{25}(16a\omega^2 - 5g)$.

(10)

The string will break if the tension in it reaches a magnitude of $4mg$.

The time for the particle to make one revolution is S.

(b) Show that

$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$$

(5)

(c) State how in your calculations you have used the assumption that the string is light.

(1)

Question 6 continued

Question 6 continued

(Total for Question 6 is 16 marks)

7.

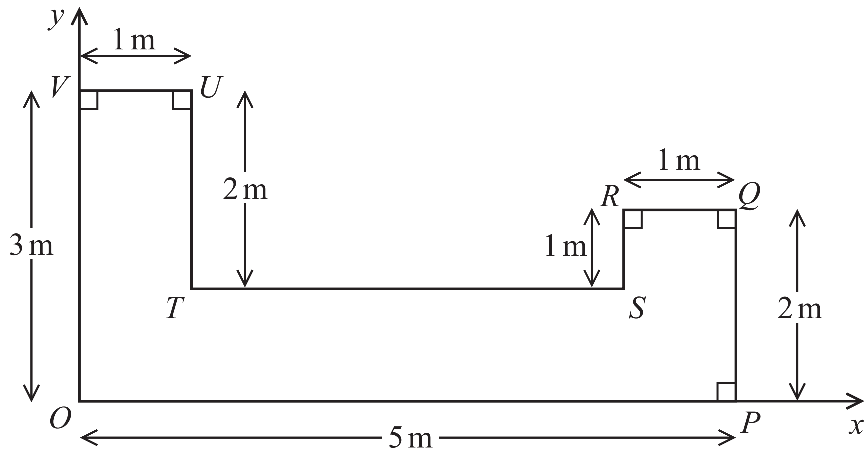

Figure 1

Figure 1 shows the shape and dimensions of a template *OPQRSTUV* made from thin uniform metal.
$OP = 5$ m, $PQ = 2$ m, $QR = 1$ m, $RS = 1$ m, $TU = 2$ m, $UV = 1$ m, $VO = 3$ m.
Figure 1 also shows a coordinate system with O as origin and the x-axis and y-axis along OP and OV respectively. The unit of length on both axes is the metre.

The centre of mass of the template has coordinates $(\bar{x}, \bar{y})$.

(a) (i) Show that $\bar{y} = 1$

(ii) Find the value of $\bar{x}$.

(7)

A new design requires the template to have its centre of mass at the point (2.5, 1). In order to achieve this, two circular discs, each of radius r metres, are removed from the template which is shown in Figure 1, to form a new template L. The centre of the first disc is (0.5, 0.5) and the centre of the second disc is (0.5, a) where a is a constant.

(b) Find the value of r.

(4)

(c) (i) Explain how symmetry can be used to find the value of a.

(ii) Find the value of a.

(2)

The template L is now freely suspended from the point U and hangs in equilibrium.

(d) Find the size of the angle between the line TU and the horizontal.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option J

Further Mechanics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \quad => \quad v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \quad => \quad u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh => h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
		(10 marks)	

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton's Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton's Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
2(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5g \times 20$	A1	1.1b
	$= 58.25 = 58$ (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
		(6 marks)	

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
3(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
		(8 marks)	

Notes:

(a)
B1: For force = 30 seen
M1: For use of $P = Fv$
A1ft: For 120 (W), follow through on their '30'

(b)
M1: For resolving parallel to the slope with correct no. of terms and 60*g* resolved
A1: For a correct equation
A1: For $F = 70$
M1: For use of $P = Fv$
A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
4(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
		(16 marks)	

Notes:

(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton's Law of Restitution, with e on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for v
A1: For a correct expression for v

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of e

(c)
M1: For solving their equations for w
A1: For the given answer

Question 4 notes continued:	
(d)	
M1:	For substituting $e = \frac{5}{9}$ into their v and w
A1:	For correct expressions for v and w
M1:	For use of Newton's Law of Restitution, with e on the correct side
M1:	For use of appropriate inequality
A1:	For a correct inequality
A1:	For a correct range

Further Mechanics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5 (a)	Multiply out and differentiate wrt t	M1	1.1b
	$v = 3t^2 - 16t + 20 \Rightarrow a = 6t - 16$	A1	1.1b
		(2)	
(b)	Multiply out and integrate wrt t	M1	1.1b
	$s = \int 3t^2 - 16t + 20\mathrm{d}t = t^3 - 8t^2 + 20t(+C)$	A1	1.1b
	$t = 0, s = 0 \Rightarrow C = 0$ $t = 2, s = 8 - 32 + 40 = 16$	A1	1.1b
		(3)	
(c)	$s = 0 \Rightarrow t^3 - 8t^2 + 20t = 0$ and $t \neq 0 \Rightarrow t^2 - 8t + 20 = 0$	M1	2.1
	Explanation to show that $t^2 - 8t + 20 > 0$ for all t.	M1	2.4
	So $s = 0$ has no non-zero solutions, so s is never zero again, so never returns to O *	A1*	3.2a
		(3)	
			(8 marks)

Notes:

(a)
M1: For multiplying out and differentiating (powers decreasing by 1)
A1: For a correct expression for a

(b)
M1: For multiplying out and integrating (powers increasing by 1)
A1: For a correct expression for s with or without C
A1: For $C = 0$ and correct final answer

(c)
M1: For equating their s to 0 and producing a quadratic
M1: For clear explanation that $t^2 - 8t + 20 > 0$ for all t (e.g. completing the square or another complete method)
A1*: For a correct conclusion in context

Question	Scheme	Marks	AOs
6(a)	$\cos\alpha = \frac{4}{5}$ or $\sin\alpha = \frac{3}{5}$	B1	1.1b
	$r = 4a\sin\alpha$	B1	1.1b
	Resolving vertically	M1	3.1b
	$T_1\cos\alpha - T_2\sin\alpha = mg$	A1	1.1b
	Resolving horizontally	M1	3.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	Solving for either tension	M1	2.1
	$T_1 = \frac{4m}{25}(9a\omega^2 + 5g)$ *	A1*	1.1b
	$T_2 = \frac{3m}{25}(16a\omega^2 - 5g)$ *	A1*	1.1b
		(10)	
(b)	$\frac{4m}{25}(9a\omega^2 + 5g) < 4mg$	M1	2.1
	$\frac{3m}{25}(16a\omega^2 - 5g) > 0$	M1	2.1
	$\omega > \sqrt{\frac{5g}{16a}}$ or $\omega < \sqrt{\frac{20g}{9a}}$	A1	2.2a
	$S = \frac{2\pi}{\omega}$	M1	1.1b
	$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$ *	A1*	1.1b
		(5)	
(c)	String being light implies that the tension is constant in both portions of the string	B1	3.5b
		(1)	
		(16 marks)	

Notes:

(a)

B1: For correct trig. ratio seen
B1: For a correct radius expression seen
M1: For resolving vertically with correct no. of terms and tensions resolved
A1: For a correct equation
M1: For resolving horizontally with correct no. of terms and tensions resolved
A1A1: For a correct equation
M1: For solving their two equations to find either tension
A1*: For the given answer
A1*: For the given answer

Question 6 notes continued:	
(b)	
M1:	For use of $T_1 < 4mg$
M1:	For using $T_2 > 0$
A1:	For a correct inequality (either) for ω
M1:	For use of $S = \frac{2\pi}{\omega}$ with either critical value
A1*:	For given answer
(c)	
B1:	For a clear explanation

Question	Scheme	Marks	AOs
7(a)	Rel. Mass: 2 5 1 8	B1	1.2
	y : 2 0.5 1.5 $\bar{y}$	B1	1.2
	x : 0.5 2.5 4.5 $\bar{x}$	B1	1.2
	$(2 \times 2) + (5 \times 0.5) + (1 \times 1.5) = 8\bar{y}$	M1	2.1
	$\bar{y} = 1$ *	A1*	1.1b
	$(2 \times 0.5) + (5 \times 2.5) + (1 \times 4.5) = 8\bar{x}$	M1	2.1
	$\bar{x} = 2.25$	A1	1.1b
		(7)	
(b)	Use of correct strategy to solve the problem by use of 'moments equation'	M1	3.1b
	$(8 \times 2.25) - (2\pi r^2 \times 0.5) = (8 - 2\pi r^2)2.5$	A1ft	1.1b
	Solving for r	M1	1.1b
	$r = \frac{1}{\sqrt{2\pi}} = 0.399$	A1	1.1b
		(4)	
(c)	Since $\bar{y}$ for original plate is 1, holes must be symmetrically placed about the line $y = 1$	B1	2.4
	$a = 1.5$	B1	2.2a
		(2)	
(d)	Use of tan from an appropriate triangle	M1	1.1a
	$\tan\alpha = \frac{2}{1.5} = \frac{4}{3}$	A1ft	1.1b
	$\alpha = 53.1°$	A1	1.1b
		(3)	
		(16 marks)	

Notes:

(a)
B1: For correct relative masses
B1: For correct y values
B1: For correct x values
M1: For a moments equation, correct no. of terms, condone sign errors
A1*: For a correct given answer (1)
M1: For a moments equation, correct no. of terms
A1: For 2.25

(b)
M1: For a moments equation, correct no. of terms, condone sign errors
A1ft: For a correct equation, follow through on their $\bar{x}$
M1: For solving for r
A1: For 0.399 or 0.40

Question 7 notes continued:	
(c)	
B1:	For consideration of symmetry about $y = 1$
B1:	For $a = 1.5$
(d)	
M1:	For use of tan from an appropriate triangle
A1ft:	For a correct equation, follow through on their a
A1:	For a correct angle